Fitting the IS–MP–PC Model

Comparative Statics, Calibration, Structural Decomposition, Dynamics and Simulation

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Outline

1. Motivation
2. Calibration
   - Optimization Framework
   - Worked Numerical Example
   - What Could Go Wrong?
3. Inducing Structural
   - From Equilibrium to Components
   - Accounting Normalization
4. Dynamic Effects
   - Shocks and Impulse Responses
   - Expectations and Persistence
5. Simulation

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IS–MP–PC Model

$$\text{IS:} \quad y_t = y^* - \alpha(r_t - r^*) + \epsilon_t^{IS}$$

$$\text{MP:} \quad r_t = r^* + \beta(\pi_t - \pi^*)$$

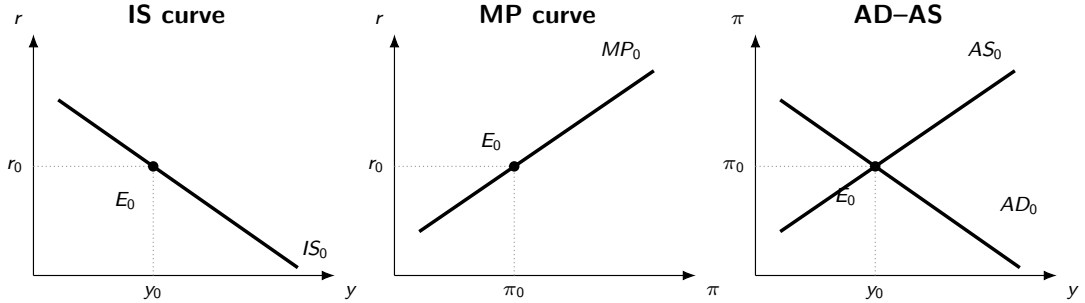
$$\text{PC:} \quad \pi_t = E_t[\pi_{t+1}] + \delta(y_t - y^*) + \epsilon_t^{PC}$$

Model structure

Three *simultaneous relationships*; three endogenous variables: y_t, π_t, r_t. Shocks and expectations are treated as exogenous within each period.

Assume $\alpha, \beta > 0$ and $\delta \geq 0$. For an isolated contemporaneous shock, hold all other inputs fixed.

IS–MP–PC System



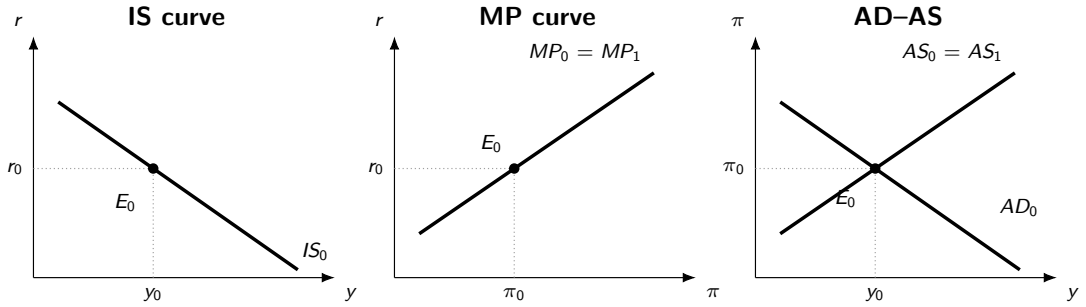
Reduced-Form Equilibria

$$y_t^{eq} = y^* + \left[\delta + \frac{1}{\alpha\beta}\right]^{-1}\left[\pi^* + \frac{\epsilon_t^{IS}}{\alpha\beta} - E_t[\pi_{t+1}] - \epsilon_t^{PC}\right]$$

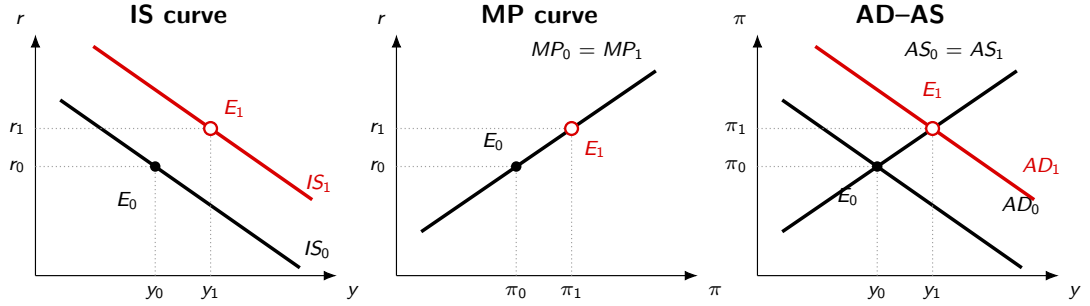
$$\pi_t^{eq} = \frac{\delta}{\delta + \frac{1}{\alpha\beta}}\pi^* + \left[E_t[\pi_{t+1}] + \epsilon_t^{PC}\right]\left(1 - \frac{\delta}{\delta + \frac{1}{\alpha\beta}}\right) + \frac{\delta}{1+\alpha\beta\delta}\epsilon_t^{IS}$$

$$r_t^{eq} = r^* - \frac{\beta}{1+\delta\alpha\beta}\pi^* + \frac{\beta}{1+\delta\alpha\beta}\left[E_t[\pi_{t+1}] + \epsilon_t^{PC}\right] + \frac{\beta\delta}{1+\alpha\beta\delta}\epsilon_t^{IS}$$

Response to a Positive IS Shock

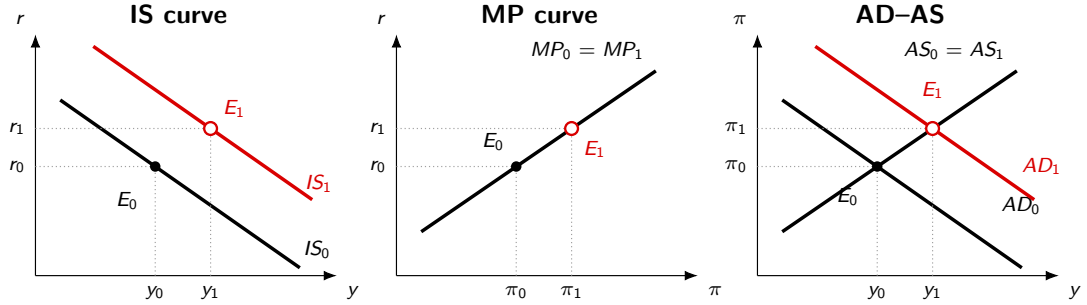


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Direction: $\epsilon_t^{IS} \uparrow$ shifts IS outward and AD rightward; MP and AS stay put.

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But how large are the movements in y, π, and r?

Analytical Approach: Parameter Sensitivity

Q: Does a larger δ amplify or mute the output response?

For a change in the IS shock, define the **output response**:

$$PI_y(\delta) \equiv \Delta y_t^{eq} = \frac{\Delta \epsilon_t^{IS}}{1 + \alpha\beta\delta}.$$

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Holding the shock and other parameters fixed, differentiate with respect to δ:

$$\boxed{\frac{\partial PI_y}{\partial \delta} = -\frac{\alpha\beta\, \Delta \epsilon_t^{IS}}{(1 + \alpha\beta\delta)^2}}$$

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For a **positive IS shock**, $\Delta \epsilon_t^{IS} > 0$, with $\alpha, \beta > 0$:

$$\frac{\partial PI_y}{\partial \delta} < 0.$$

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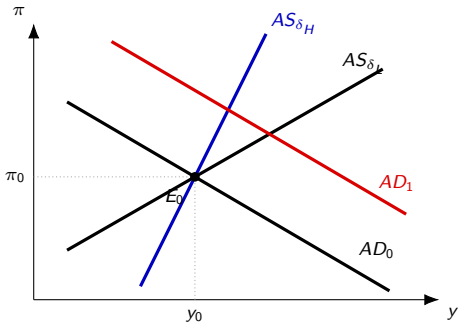
Conclusion: A larger δ mutes the increase in equilibrium output caused by the same positive IS shock.

Economic Intuition

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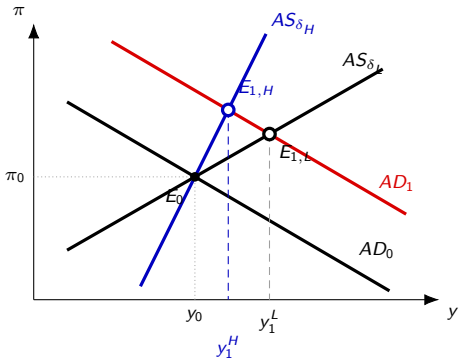
Q: Why does a larger δ reduce the output response?



- A larger δ means a given positive **output gap generates greater inflationary pressure**:
  - Firms may adjust prices more readily.
  - Marginal costs may rise more sharply near capacity.
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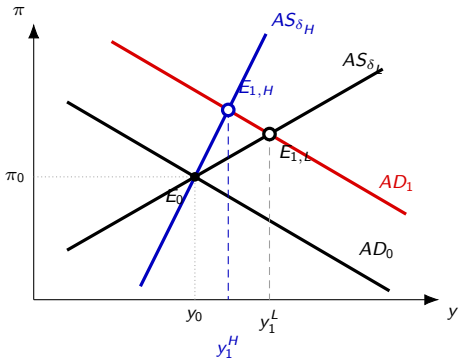
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Output response: $0 < \Delta y_H < \Delta y_L$.

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  - Marginal costs may rise more sharply near capacity.
- The **same positive IS shock** shifts AD rightward in both economies.
- A steeper AS curve produces a **larger rise in inflation**.
- MP responds with a **larger rise in the real interest rate**, curbing interest-sensitive spending and muting output.

Analytical Approach

Q: How do inflation and the real rate respond?

A useful substitution technique

Anchor on output, then substitute into PC and MP. This is *not* a chronology: all three equations hold in the same-period equilibrium.

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$$\Delta y^{eq} = \frac{1}{1+\alpha\beta\delta}\Delta\epsilon^{IS},$$

$$\Delta\pi^{eq} = \delta\,\Delta y^{eq} = \frac{\delta}{1+\alpha\beta\delta}\Delta\epsilon^{IS},$$

$$\Delta r^{eq} = \beta\,\Delta\pi^{eq} = \frac{\beta\delta}{1+\alpha\beta\delta}\Delta\epsilon^{IS}$$

Numerical Approach: Perturb and Compare

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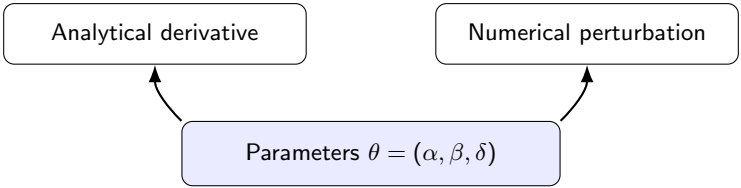
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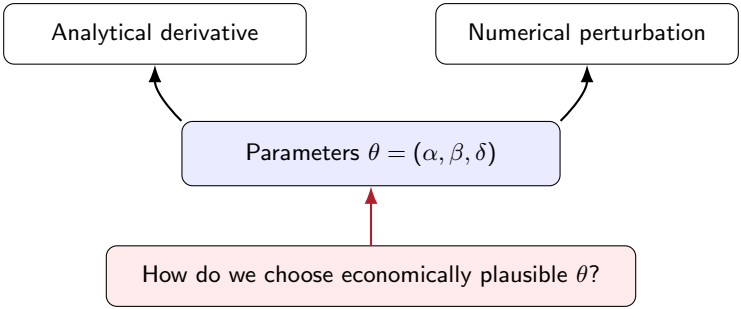
Why both methods?

- Numerical: Quantify for specific calibration
- Analytical: Reveals general properties of the model

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Key consideration

Different targets reveal different information about θ. A single steady-state observation cannot identify all three parameters.

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Our exercise: choose parameters so that modeled macro outcomes align with selected targets.

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$$\alpha > 0, \qquad \beta > 0, \qquad \delta \geq 0$$

Calibrating the IS–MP–PC Model

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Our example: Match output, inflation, and interest rates across scenarios $s = 1, \ldots, S$:

$$\begin{aligned} L(\theta) = \sum_{s=1}^{S} \big[& w_y \big(y_s^{eq}(\theta) - y_s^{target} \big)^2 \\ & + w_\pi \big(\pi_s^{eq}(\theta) - \pi_s^{target} \big)^2 \\ & + w_r \big(r_s^{eq}(\theta) - r_s^{target} \big)^2 \big]. \end{aligned}$$

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- Weights account for units, scales, and priorities.
- Targets must be informative: a single steady-state equilibrium does *not* identify α, β, and δ.

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Suppose we observe an economy with elevated expected inflation. For this *illustrative exercise* (not empirical data), take the following reference values and exogenous inputs:

$$y^* = 100, \quad \pi^* = 2, \quad r^* = 0.5, \quad E_t[\pi_{t+1}] = 3, \quad \epsilon_t^{IS} = \epsilon_t^{PC} = 0.$$

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Observed quantity	**Target level**	**Units**
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π^{eq}	2.7143	Percent
r^{eq}	0.8571	Percent

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Model predictions under these conditions

$$\widehat{y}^{eq} = 100 - \frac{\alpha\beta}{D}, \quad \widehat{\pi}^{eq} = 3 - \frac{\alpha\beta\delta}{D}, \quad \widehat{r}^{eq} = 0.5 + \frac{\beta}{D}, \qquad D = 1 + \alpha\beta\delta.$$

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Our objective: Choose $\theta = (\alpha, \beta, \delta)$ to reproduce the observed levels.

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Q: How closely does an initial parameterization reproduce the observed levels?

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	Target	Prediction	Squared error
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π^{eq}	2.7143	2.8333	0.01417
r^{eq}	0.8571	0.9167	0.00354

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Q: Can we improve the fit by changing the parameters?

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y^{eq}	99.2857	99.4231	0.01887
π^{eq}	2.7143	2.7692	0.00302
r^{eq}	0.8571	0.8846	0.00075

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- The loss has decreased from 0.10629 to 0.02264.
- The fit has improved, but discrepancies remain.

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- Starting values, objective weights, algorithms, and constraints affect the search and its outcome.

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Two distinct inputs

Parameters describe how the economy *transmits* disturbances; shock magnitudes, persistence, and distributions must be specified or calibrated separately.

Outline

1. Motivation
2. Calibration
   - Optimization Framework
   - Worked Numerical Example
   - What Could Go Wrong?
3. Inducing Structural
4. Dynamic Effects
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Interpretation

Calibration gives us a *useful working model* conditional on the targets and assumptions. A good fit does not prove that the structural parameters have been uniquely identified.

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Remedy: Normalize or weight the errors

$$L(\theta) = w_y(\widehat{y} - y^{target})^2 + w_\pi(\widehat{\pi} - \pi^{target})^2 + w_r(\widehat{r} - r^{target})^2.$$

$w_y, w_\pi, w_r > 0$ are chosen to account for measurement scales, typical variability, or economic priorities.

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Same inputs: $y^* = 100$, $\pi^* = 2$, $r^* = 0.5$, $E_t[\pi_{t+1}] = 3$, $\epsilon^{IS} = \epsilon^{PC} = 0$.

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Diagnosis: The targets do not separately identify all parameters

Fitting y and π identifies $\delta \approx 0.4$ and $\alpha\beta \approx 1$, but not α and β separately. The observed real rate instead implies

$$\beta = \frac{r - r^*}{\pi - \pi^*} = \frac{0.6500 - 0.5}{2.7143 - 2} \approx \boxed{0.21} \neq 0.5.$$

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Remedy: Add r as a calibration target and refit jointly. Here $\theta \approx (4.76, 0.21, 0.40)$ fits all three levels.

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Remedies: Recheck data and assumptions, reconsider r^*, or modify the MP specification if economically justified.

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Remedy: Include informative variation across periods or scenarios, such as observed responses to identified disturbances.

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Structural Relations versus Reduced Form

Q: What do the two terms mean?

Structural equations in the system

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A separate task

After solving these equilibrium quantities, we can use them to *induce* estimates of consumption, investment and other GDP components. This need not imply a separately solved structural expenditure equilibrium.

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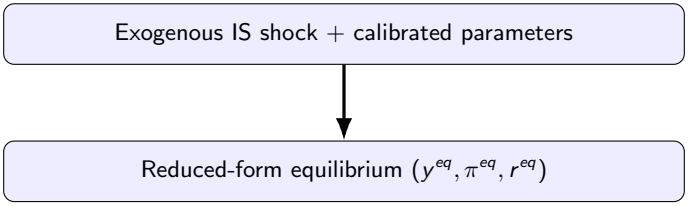
Our Approach

IS, MP and PC are structural; solving them produces the reduced form. We use its equilibrium to *induce* expenditure components such as C and I.

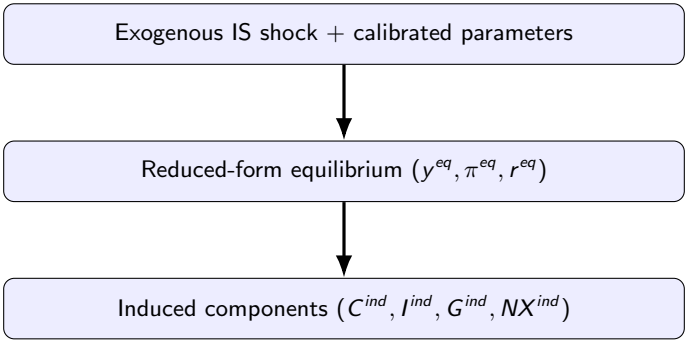
Our Direction of Analysis

Exogenous IS shock + calibrated parameters

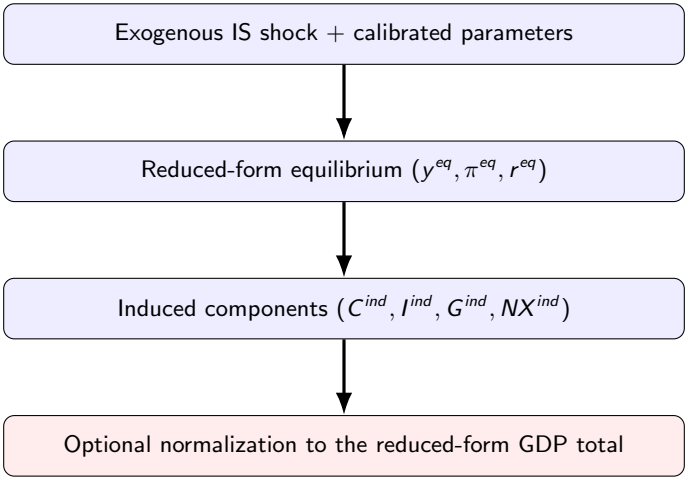
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Two distinct notions of GDP

y_t^{eq}: the pre-determined IS–MP–PC macro equilibrium.
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y_t^{ind} need not equal y_t^{eq} under an arbitrary component calibration.

An IS Shock: Macro Outcomes to Components

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- The net effect on induced consumption is *not predetermined* without the structural coefficients c_1 and c_2.

Why Might the IS Shock Occur?

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Keep the two exercises distinct

Before equilibrium: translate the underlying development into an IS shock.
After equilibrium: use y^{eq}, r^{eq} to induce $C^{ind}, I^{ind}, \ldots$.

Outline

1. Motivation
2. Calibration
3. Inducing Structural
   - From Equilibrium to Components
   - Accounting Normalization
4. Dynamic Effects
5. Simulation

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This discrepancy is possible: the induced components are not an independently solved GDP accounting system.

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Interpretation

This is an *accounting allocation by construction*, not an independent equilibrium condition. Because φ_t varies, normalized components can change even when their induced levels do not.

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	Before	After
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Reduced-form GDP y^{eq}	100.00	100.00
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The problem

Consumption changes, but macroeconomic GDP does not: T does *not* enter our reduced-form equilibrium equations.

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Our computational direction

$$\text{Structural event } \Delta T > 0 \longrightarrow \text{analyst-calibrated } \Delta \epsilon^{IS} < 0$$

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The analyst supplies the mapping from the observed event to the model shock.

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For this separate illustration, take $\alpha = 1$, $\beta = 0.5$, $\delta = 0.25$. The *equilibrium output response*:

$$\Delta y_A^{eq} = \frac{\Delta \epsilon_A^{IS}}{1 + \alpha\beta\delta} = \frac{-0.37264}{1 + 1(0.5)(0.25)} \approx \boxed{-0.33123}.$$

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Attributing the entire GDP decline to this IS shock, with other shocks, expectations and parameters held fixed, requires $\Delta \epsilon^{IS} \approx -0.41922$.

From the IS Shock Back to Structural Components

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Use these *new* macro outcomes **and** the higher tax level in the structural-component equations:

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Remember the direction

Observed structural development $\longrightarrow$ calibrated IS shock $\longrightarrow$ reduced-form equilibrium $\longrightarrow$ induced components (then optional normalization).

Outline

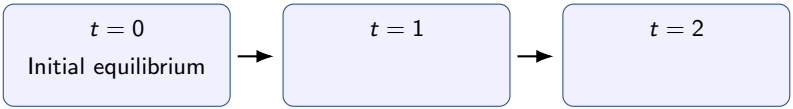
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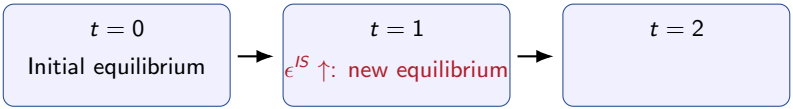
From Static to Dynamic

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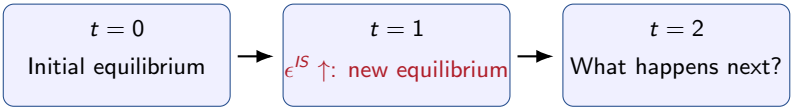
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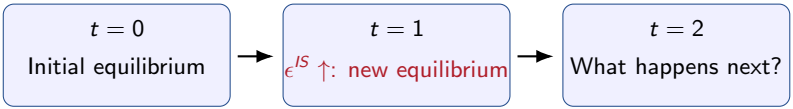
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Q: Does the shock itself persist? Even if it does not, could its effects persist?

Two Sources of Dynamics

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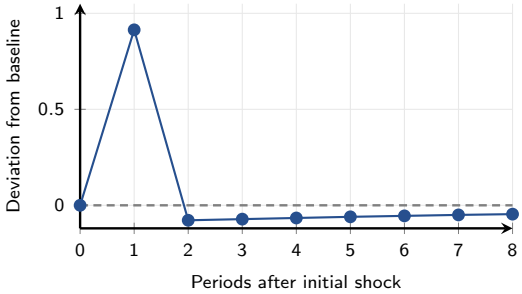
Shock duration $\neq$ Response duration

Impulse Response Function

Tool to visualize the dynamics of our macro model

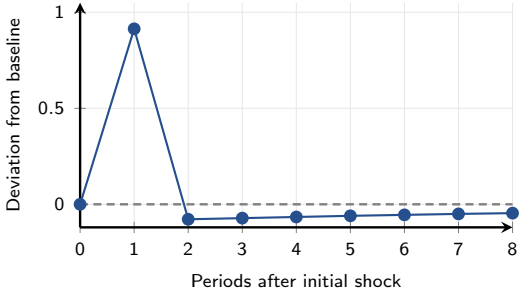
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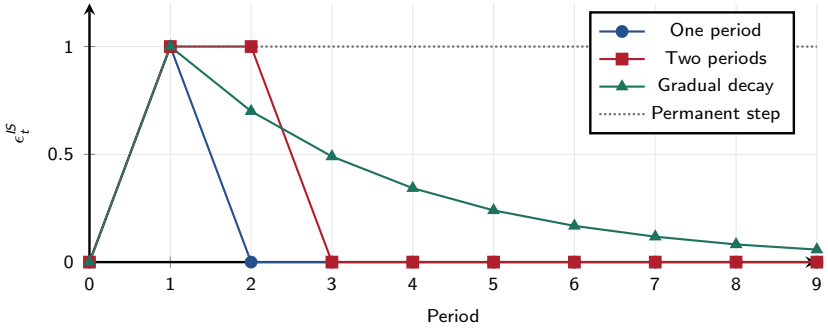
Tool to visualize the dynamics of our macro model



- x: time since shock
- y: response *relative to a baseline*
- initial impact, sign reversal, persistence, and eventual convergence

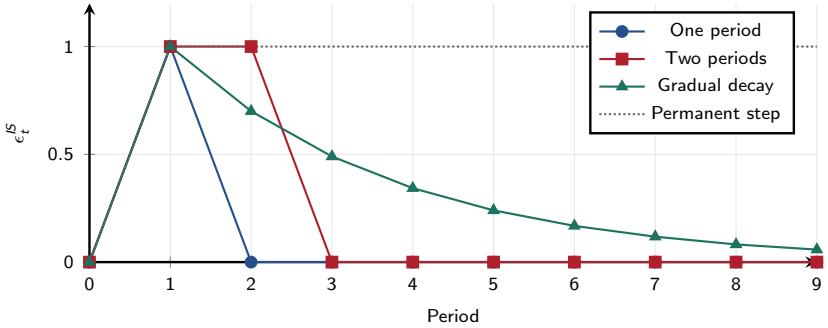
Different Shock Paths

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Each path is an *assumption about the shock process*. We feed those values into the model one period at a time.

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Case B: adaptive expectations

If realized inflation in period 1 influences expected inflation in period 2, the period-2 PC changes even though the IS shock has vanished. The period-2 outcomes may differ from baseline.

Outline

1. Motivation
2. Calibration
3. Inducing Structural
4. Dynamic Effects
   - Shocks and Impulse Responses
   - Expectations and Persistence
5. Simulation

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- w_j: importance attached to lag j.
- The expectation is about π_{t+1}, but under the specified classroom rule it is formed from past observed inflation through $t-1$.

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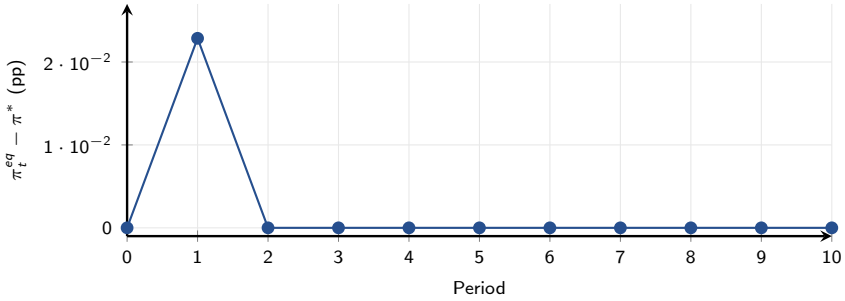
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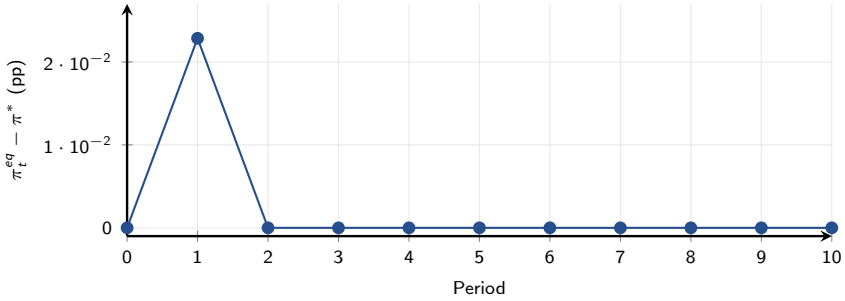
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Inflation rises only when the shock is active; with no dynamic expectations link, it returns to target immediately afterward.

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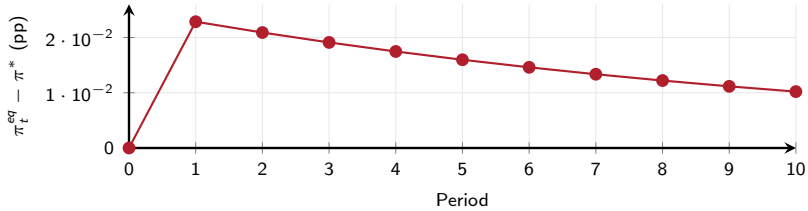
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Q: How does a longer memory alter the time profile?

$$\boxed{E_t[\pi_{t+1}] = \tfrac{1}{2}\pi_{t-1} + \tfrac{1}{2}\pi_{t-2}}$$

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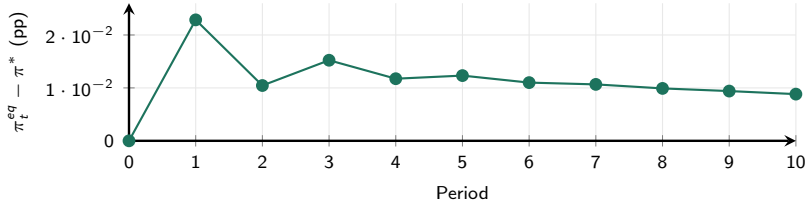
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- Expectations incorporate both the initial inflation spike and the pre-shock inflation rate.

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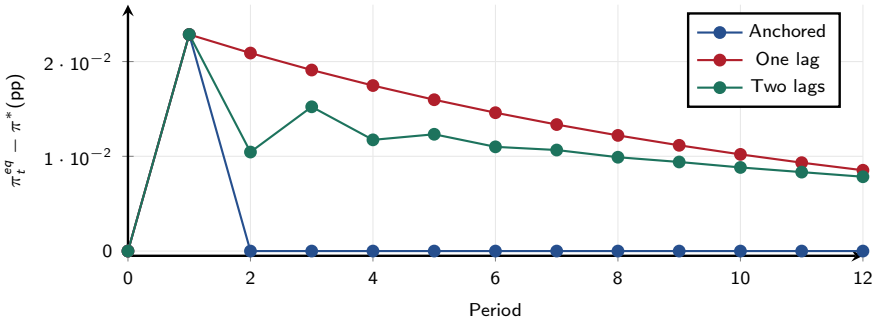
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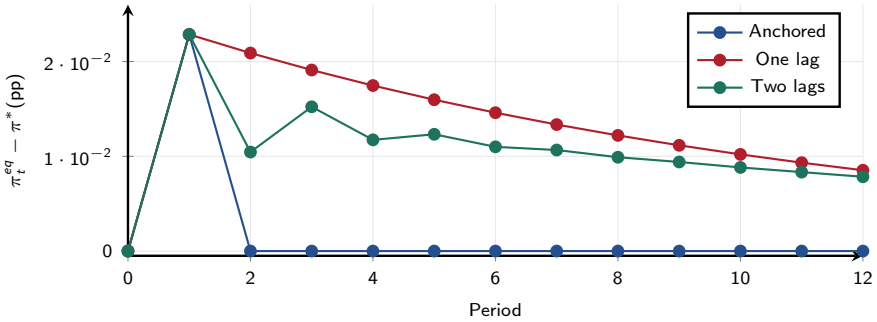
The two-lag memory creates alternating rises and falls. But $\rho < 1$ dampens the feedback, so inflation eventually converges toward its target.

Three Expectation Rules, One Initial Shock

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A two-lag rule can dampen the initial carryover yet leave a longer tail. The persistence ranking need not be identical at every horizon.

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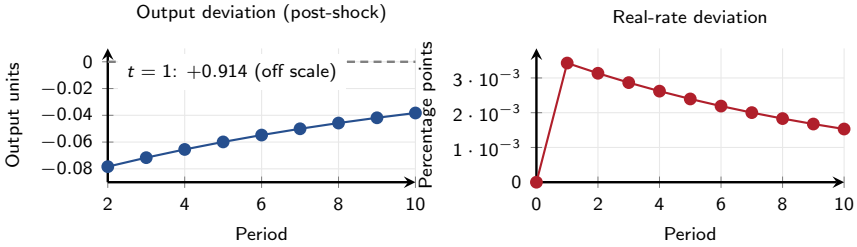
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- Larger $\alpha\beta\delta$ makes ρ smaller: inflation deviations decay more quickly in this simplified case.
- Long-run convergence is *conditional* on the shock process and expectations mechanism; a permanent shock need not have the same limit.

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One model + many plausible shock paths $\longrightarrow$ distribution of outcomes.

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5. Summarize a forecast distribution or a stress-test outcome.

Mean versus Volatility of Shocks

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Mean versus Volatility of Shocks

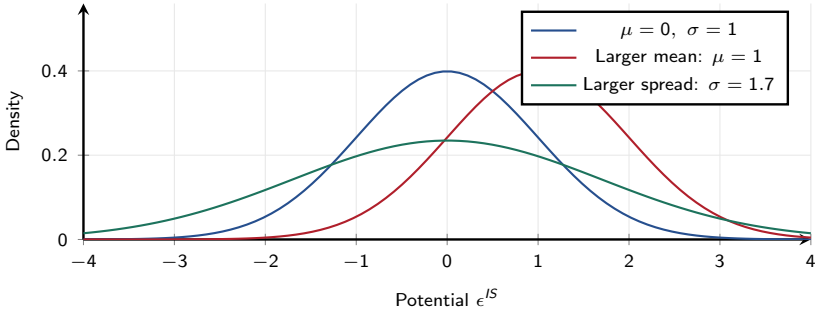
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Higher mean $\Rightarrow$ more positive shocks on average, not necessarily larger *absolute* shocks.

Higher $\sigma \Rightarrow$ more dispersion and larger tail realizations.

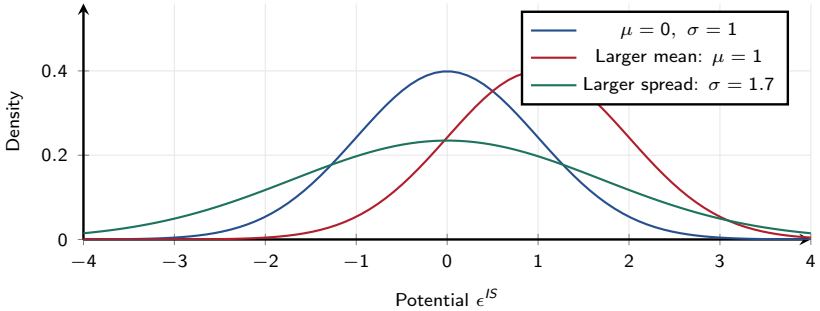
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Mean versus Volatility of Shocks

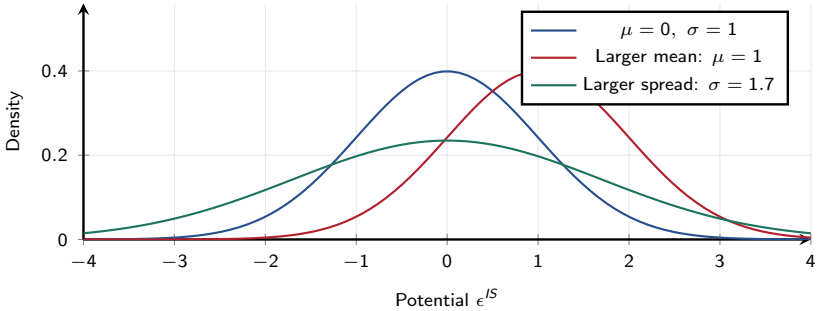
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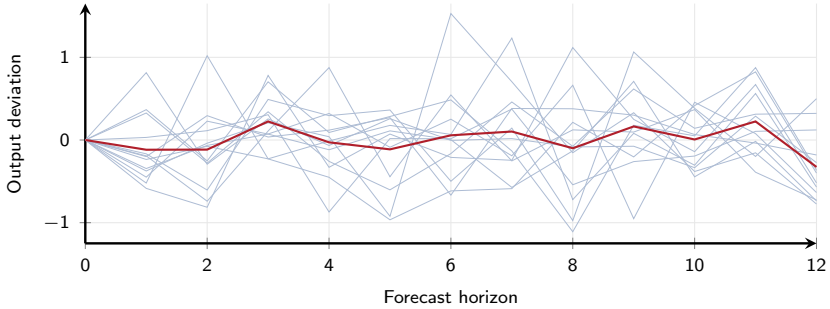
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Simulation: From Paths to a Distribution

Q: What do many draws tell us that a single scenario cannot?



Illustrative simulated paths (not actual macro forecasts). Report distributions, quantiles, and adverse tails rather than only an average path.

Choosing the Shock Distribution

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- **Stress scenarios:** Deliberately choose tail magnitudes/durations rather than relying solely on normal draws.

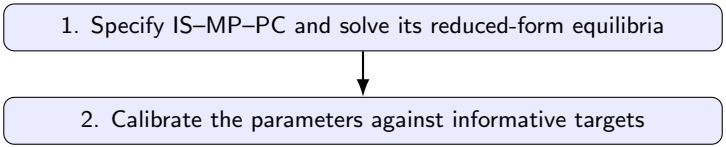
Putting the Lecture Together

Q: What is the workflow for using a calibrated IS–MP–PC model?

1. Specify IS–MP–PC and solve its reduced-form equilibria

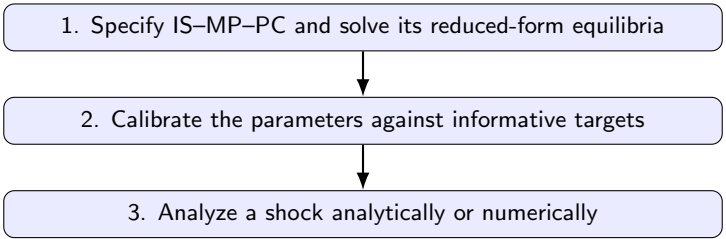
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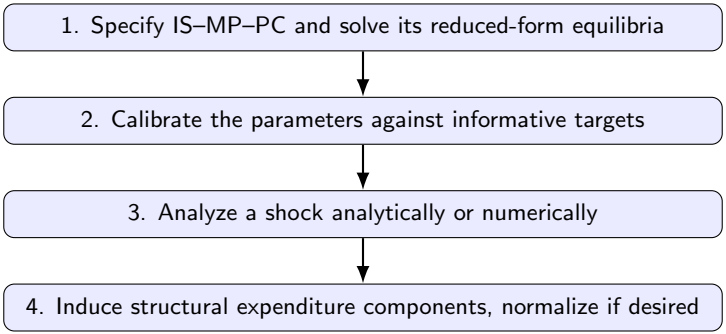
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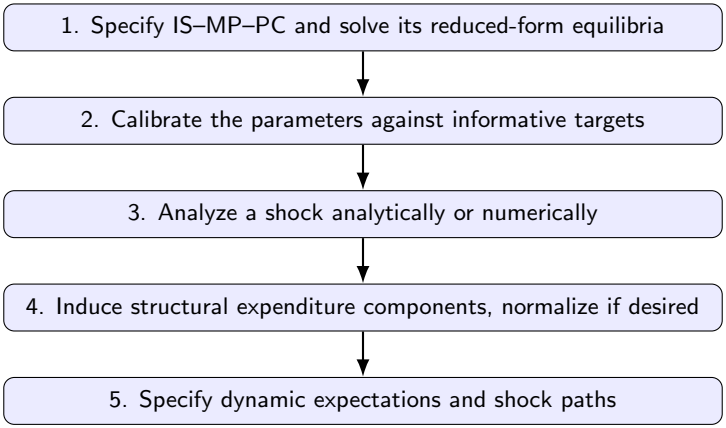
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