Potential Outcomes

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Potential Outcomes - Summary

- Our object of interest is the treatment effect; impact of a policy on an outcome
- Fundamental problem of causality: we can't see an individual with and without treatment
- So we resort to estimates averages (ATE, ATT, ATC, CATE)
- Estimates of treatment effects based upon observed data usually contain an unobserved selection bias

Housekeeping

- Throughout the slide deck you will see "Q", which indicates a question to you, the reader
- You will also see "A", which indicates the associated answer
- It is generally most efficient to learn this material through active participation. Whenever you encounter a "Q", be sure to try and develop your answer before turning the page to the provided "A" answer
- I recommend reading through these slides before engaging in associated coding exercises
- This slide deck is part of a broader curriculum in causal estimation and inference
- Suggested pre-requisite: Working knowledge of (un)conditional expectations

Outline

1. Motivation - Case Study

2. Potential Outcomes Framework
   - Potential Outcomes
   - Treatment Effects
   - Selection Bias

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Does Insurance Improve Health?



Image created with Bing Image Creator
The following is adapted from the book Mastering 'Metrics

Meet Khuzdar

Meet Khuzdar



- He has health insurance
- He is known to be relatively weak (e.g. poor constitution, gets sick often, etc..)
- We want to know the impact health insurance has on his health as measured by NHIS health index score (higher is better)

Fundamental Problem of Causal Inference

Q: Why is it difficult to estimate the impact of insurance for Khuzdar?

Fundamental Problem of Causal Inference

Q: Why is it difficult to estimate the impact of insurance for Khuzdar?

A: We don't see him with and without insurance.







Earth 1



Earth 0

Viewing Khuzdar Over Time With(out) Insurance

Q: Why is viewing Khuzdar over time not suitable?



2018

2019



Khuzdar has health insurance

2020

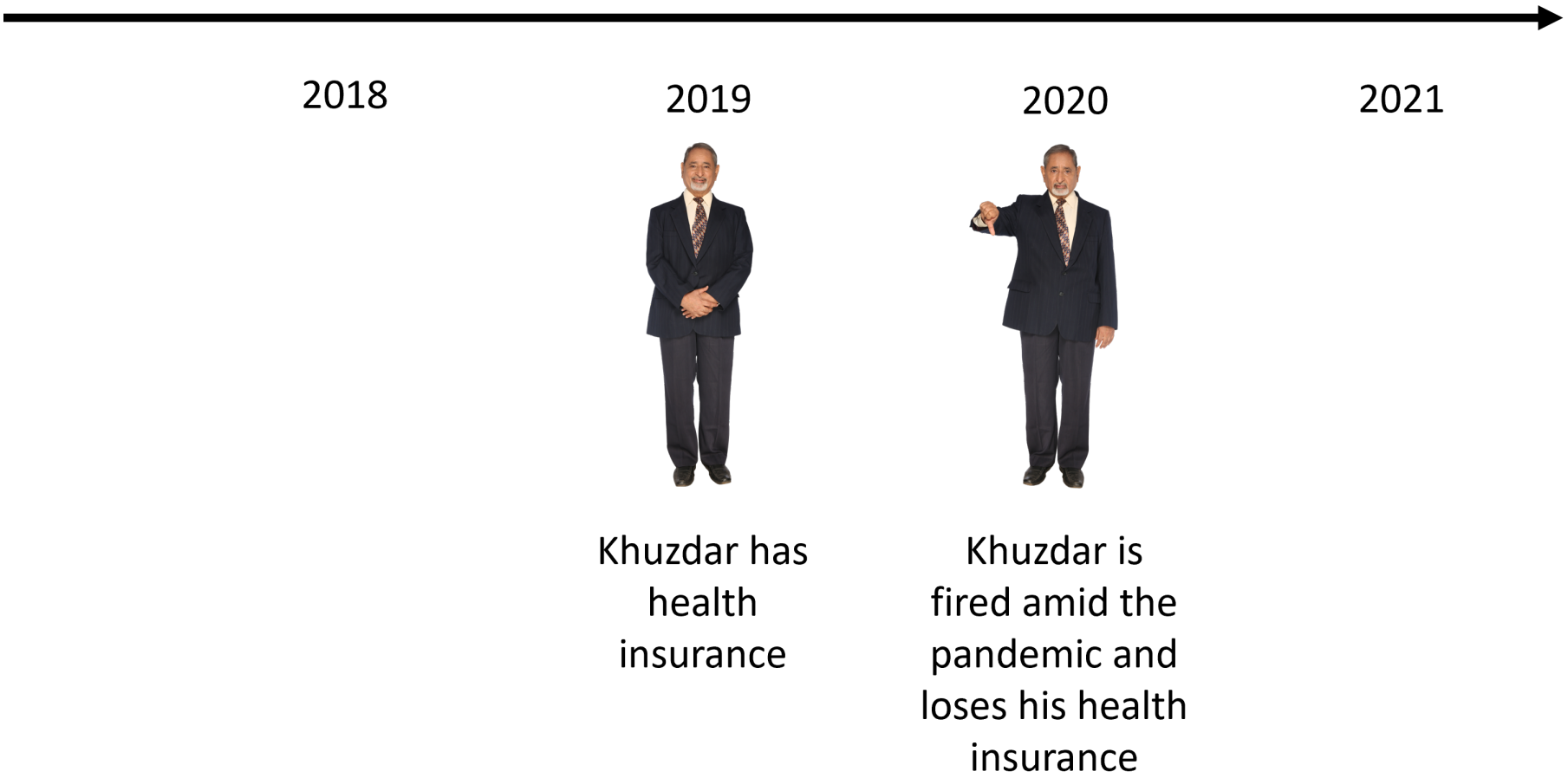


Khuzdar is fired amid the pandemic and loses his health insurance

2021

Viewing Khuzdar Over Time With(out) Insurance

Q: Why is viewing Khuzdar over time not suitable?



A: Khuzdar really isn't the same person in 2020. Lots of things may have changed (confounders), that also impact his health. All is not "c.p.".

Meet Maria



- She does not have health insurance
- She is known to be relatively strong (e.g. good constitution, doesn't gets sick often, etc..)

Comparing Khuzdar and Maria

Q: Why shouldn't we use Maria to determine impact of insurance on Khuzdar's health?

Khuzdar



VS

Maria



Comparing Khuzdar and Maria

Q: Why shouldn't we use Maria to determine impact of insurance on Khuzdar's health?



A: Maria is stronger than Khuzdar. Comparing their health isn't fair. We don't know if we are capturing the impact of insurance or inherent strength.

Path Forward

- In order to elicit the policy impact we'll generally need more individuals to compare
- We usually also need data on how those individuals differ
- We also need a framework for distinguishing the impact of insurance from other innate reasons

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Jargon Alert: Confounders are sometimes called covariates or control variables

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Jargon Alert: Confounders are sometimes called covariates or control variables

Jargon Alert: The term "Control" is used in myriad contexts throughout the Causality literature. In the setting outlined above we can think of the "Control Group" as the "Untreated Group"

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- Outcome Y: Health (e.g. NHIS health index score)
- Treatment D:

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- Treated:

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Potential Outcomes Notation

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Y_{1i} : Potential outcome of Y for individual i in treated state

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$Y_{0i}|D_i = 1$: Potential outcome of Y for individual i in the untreated state, given that i is observed to be treated

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Khuzdar's Health

Q: Can you write notation for Khuzdar's potential outcomes on Earth 1 and Earth 0?

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A:



$Y_{1i}|D_i = 1$



$Y_{0i}|D_i = 1$

Maria's Health

Q: Can you write notation for Maria's potential outcomes on Earth 1 and Earth 0?

Maria's Health

Q: Can you write notation for Maria's potential outcomes on Earth 1 and Earth 0?

A:



$Y_{1i}|D_i = 0$



$Y_{0i}|D_i = 0$

Potential Outcomes Notation in Our Case

$$
\begin{aligned}
Y_{1i}: \\
Y_{0i}: \\
D_i = 1: \\
Y_{0i}|D_i = 1: \\
Y_{0i}|D_i = 0: \\
Y_{1i}|D_i = 1: \\
Y_{1i}|D_i = 0:
\end{aligned}
$$

Potential Outcomes Notation in Our Case

$$
\begin{aligned}
Y_{1i} &: \text{Health for individual i with insurance} \\
Y_{0i} &: \\
D_i = 1 &: \\
Y_{0i}|D_i = 1 &: \\
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Potential Outcomes Notation in Our Case

Y_{1i} : Health for individual i with insurance

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$Y_{0i}|D_i = 1$: Khuzdar's health if he didn't have insurance; Y_{0K}

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$Y_{1i}|D_i = 1$: Khuzdar's health if he did have insurance; Y_{1K}

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$$Khuzdar : Y_{1i}|D_i = 1 \rightarrow Y_i|D_i = 1$$
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During estimation we leverage this to form our expectations

$$Khuzdar : E[Y_{1i}|D_i = 1] = E[Y_i|D_i = 1]$$
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We don't expect to see Khuzdar without insurance, nor do we see Maria with insurance.

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Treatment Effect Terminology

- Outcome Y: Object of interest
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The Ideal TE

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Q: What's the difficulty with estimating ITE?
A: This is akin to the impact of insurance for Khuzdar. Since it is based on the potential outcome for an individual it is unobservable, and some might say unobtainable without strict assumptions.

Average Treatment Effects

Since we can't observe/estimate TE for individuals, we resort to averages for groups

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- Average TE (ATE) $E[Y_{1i} - Y_{0i}]$

Notice that ATE is a weighted average of the ATT and ATC

$$ATE = Prob(D_i = 1) \times ATT + Prob(D_i = 0) \times ATC$$

My Treatment Impacting Your Outcome

Stable Unit Treatment Value Assumption (SUTVA)

- In order to identify the ATT/ATC/ATE in sample, we need to ensure that the treated group's treatment does not impact the control group.
- SUTV Assumption: Treatment received by Khuzdar does not impact Maria (or anyone else)

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- But what if there was a connection? What if Khuzdar and Maria were married. Perhaps his insurance (treatment) impacts his health, which then impacts the likelihood that Maria will also get sick
- One way to resolve the issue is to redefine the study units; e.g. treatment effect on household rather than individual

Which TE should I use?

The choice of TE is often overlooked in applied settings. This could be due to negligence on the part of the researcher, or due to the fact that these concepts overlap under strict assumptions.

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- ATC: What is the impact on public health if insurance was State granted? i.e. what if we gave (forced) Maria insurance?
- ATE: If we switch from a setting wherein no one has insurance to a setting in which everyone has insurance, what would be the impact on health?

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We can form conditional versions of the TEs

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$$CATT(x) = E[Y_{1i}|D_i = 1, X_i = x] - E[Y_{0i}|D_i = 1, X_i = x]$$
$$CATC(x) = E[Y_{1i}|D_i = 0, X_i = x] - E[Y_{0i}|D_i = 0, X_i = x]$$

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$$CATT(x) = E[Y_{1i}|D_i = 1, X_i = x] - E[Y_{0i}|D_i = 1, X_i = x]$$
$$CATC(x) = E[Y_{1i}|D_i = 0, X_i = x] - E[Y_{0i}|D_i = 0, X_i = x]$$

- CATE is the typical TE used among these 3.
- CATE can be seen as the ATE within subgroups of the population, wherein those groups are made by stratifying X.
- ATE can be seen as the weighted average of CATE across all the subgroups.
- More details when we discuss Heterogenous Treatment Effects

Interpreting CATE

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A:

$$CATE(45) = E[Y_{1i}|Age = 45] - E[Y_{0i}|Age = 45] = 2$$

Interpreting CATE

Q: Consider a confounder like Age. Can you interpret CATE(45)=2?

A:

$$CATE(45) = E[Y_{1i}|Age = 45] - E[Y_{0i}|Age = 45] = 2$$

This implies that among individuals 45yrs old, the treatment effect is 2 units. i.e. among all 45yrs in the sample, the potential health outcome of having insurance is 2 units higher than if they did not have insurance.

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Estimate vs Estimand

Q: What is the difference between a treatment effect estimate and estimand?

Notice that we can't observe the treatment effect. It must be estimated. Whenever we estimate something we need i) the "truth", 2) the formula, 3) the number

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- Estimand: ATE is the "population" concept (truth)

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- Estimator: formula/model used to create the Estimate (e.g. OLS parameter coefficient) (formula)
- Estimate: $\widehat{ATE}$ is the sample counterpart (number)

Outline

1. Motivation - Case Study

2. Potential Outcomes Framework
   - Potential Outcomes
   - Treatment Effects
   - Selection Bias

Suppose you could see the potential outcomes

Q: Can you compute and interpret ATT, ATC, ATE?

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Missing Potential Outcomes

We can't measure TE for Khuzdar nor Maria due to missing potential outcomes

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A: Having insurance reduces health by 1 unit (NHIS health index score)

Q: Why is this value not trustworthy?

A: Maria is stronger than Khuzdar, so we can't distinguish the "insurance effect" from the "Maria effect".

Distinguishing the Insurance Effect from the Maria Effect

Let's decompose what we observe

$$Y_{1K} - Y_{0M} = (Y_{1K} - Y_{0K}) + (Y_{0K} - Y_{0M})$$

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Q: Can you interpret the Selection Bias?
A: Selection Bias captures the inherent differences between Khuzdar and Maria (e.g. Maria is stronger)

Sizing the Selection Bias

Q: Can you use the table with missing outcomes to compute the ATT and selection bias?

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- ATT: $Y_{1K} - Y_{0K} = 4 - 3 = 1$
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- $\widehat{ATT} = ATT +$ Selection Bias: -1 = (1) + (-2)

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Use caution with jargon for describing the selection bias

Observed Differences = ATT + Selection Bias

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- -5 = -2 + -3 $\Rightarrow$ Estimate (-5) understates true TE (-2) by (3) units
- 4 = -2 + 6 $\Rightarrow$ Estimates (4) overstates true TE (-2) by (6) units

Size vs Sign

WORD OF CAUTION

- You cannot observe the selection bias in practice

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- Which means we will never know it's size

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- You cannot observe the selection bias in practice
- Which means we will never know it's size
- All we can do is attempt to surmise the SIGN of the bias based upon subject matter expertise. This is where combining your domain knowledge with your data science skills come in handy.

Potential Outcomes - Summary

- Our object of interest is the treatment effect; impact of a policy on an outcome
- Fundamental problem of causality: we can't see an individual with and without treatment
- So we resort to estimates averages (ATE, ATT, ATC, CATE)
- Estimates of treatment effects based upon observed data usually contain an unobserved selection bias